MP3, Grade 4

Task: Finding Equivalent Fractions

Practice standard focus: MP3. Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. . . . Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (see MP.8, Look for and express regularity in repeated reasoning). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. . . . Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

Content standard focus: 4.NF Extend understanding of fraction equivalence.

Introduction

This classroom example begins as students are trying to articulate a conjecture about the relationship between the numerator and denominator in fractions equivalent to unit fractions (MP6). Once they have articulated their conjecture, one of the students offers a representation to explain *why* the relationship they have noticed must work.

Classroom example[1]

The following is a real account by a classroom teacher. We join the class as they are discussing how to tell when a fraction is equivalent to ½.

> **Steven:** Fractions equal one-half when they can be split down the middle.
>
> **Ms. Taft:** Can you tell me more about that?
>
> **Steven:** Yes, when they are shared equally, that is like splitting it right down the middle.

[1] Adapted and reprinted with permission from *Connecting Arithmetic to Algebra: Strategies for Building Algebraic Thinking in the Elementary Grades* by Susan Jo Russell, Deborah Schifter, and Virginia Bastable. Pp. 30-31. Copyright © 2011 by TERC. Published by Heinemann, Portsmouth, NH. Students' and teacher's names are pseudonyms.

Kevin: What I think he means is that any time the numerator is half of the denominator, the fraction is equivalent to one-half.

Ms. Taft: Is there a clear and concise way I can record that?

Kevin: I know if the numerator is times 2 and you get the denominator, then the fraction equals one-half.

Ms. Taft: Is there a way I can record that without using so many words?

Lynne: You can say the numerator is the letter *n* and the denominator equals the letter *d*. Then you can write: *if n times 2 equals d, then the fraction equals one-half.*

Ms. Taft: Will that always be true? Can we apply this idea to other fractions we know?

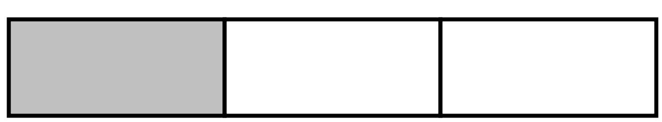
Eddie: You can do this with any numerator and denominator. You just ask yourself, "How did the numerator get to the denominator?"

Ms. Taft: So, if we think about $\frac{9}{27}$ and I ask myself how does 9 get to 27? What would you answer? Talk about this idea with your partners.

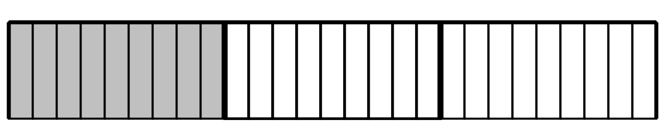
Kevin *[after some time passes during which students talk in pairs]*: I know that
$9 \times 3 = 27$ and $27 \div 3 = 9$, so I think that fraction equals one-third.

Ms. Taft: Ok, I am a little bit convinced. Can you convince me some more?

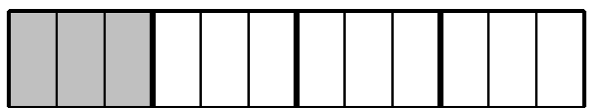
Steven: If you draw a rectangle and split it into thirds and color in one of the thirds, I can convince you. *[I follow Steven's directions; see below]*



Steven: Now, take those thirds and split them into 9 equal pieces each. You now have 27 pieces and you'll see 9 are shaded in, the same pieces that were shaded in when you drew one-third. *[Note the changes in the following figure.]*



Lucy: I know the same thing will work with one-fourth and three-twelfths. 1 times 4 is 4 and 3 times 4 is 12, so they both equal one-fourth. You can draw it the same way you did with Steven's one-third and $\frac{9}{27}$. *[The following image applies to Lucy's idea to show ¼ = 3/12.]*



The group tests a few more ideas for ¼ and 1/5.

Ms. Taft: Is there a way I can write these ideas in a quicker way?

Steven: If you do numerator times 2 equals denominator to get one-half, all you have to do is numerator times any number to get one-any numberth. Like $\frac{10}{100}$ is $\frac{1}{10}$. $\frac{5}{20}$ is ¼. It's hard to write one rule for all of these examples, but I know it will work.

The teacher decides this is a good place to stop, but she plans to continue the discussion. She has several goals in mind: to provide an opportunity for more students to contribute to the evolving argument, to see if any students come up with other representations that help develop the argument and may be accessible to different students, and to work on students' articulation of their arguments.

Commentary

Notice how the students move back and forth between particular examples and more general ideas. At first, students think about fractions equivalent to ½. Then Eddie conjectures, "You can do this with any numerator and denominator." He says, "You just ask yourself, 'How did the numerator get to the denominator?'" Eddie's question can be paraphrased as, "What can you multiply the numerator by to get the denominator?" Kevin contributes a particular example of this idea, and then Steven offers a drawing of a rectangle to show how 1/3 is equal to 9/27.

In this class, drawings, diagrams, and other representations are a central part of math instruction. Students are expected to be familiar with a range of representations, to use them as part of solving problems, and to use them to communicate their ideas to others. Steven's drawing can be seen as an argument for a single example, but Lucy shows how the same kind of representation can be used to make the argument for a different pair of equivalent fractions. Other students then contribute additional examples.

Implicit in the students' developing argument, based on their demonstration using the drawing, is that if you divide each unit fraction piece into the same number of equal pieces, then both the denominator (the total number of pieces) and the numerator (the number of pieces in each equal section) is multiplied by the same number. In Steven's drawing, the total number of pieces (3) is multiplied by 9 to become 27, and each individual section of the rectangle (1) is multiplied by 9 to become 9. Similarly, Lucy's diagram shows both 1 and 4 multiplied by 3: ¼ = 3/12.

In the elementary grades, as students become more practiced in making mathematical arguments, they often use examples involving specific numbers, but intend them to stand in for an argument about a class of numbers. Questions such as Ms. Taft's, "Can we apply this idea to other fractions we know?" and "Is there a way I can write these ideas in a quicker way?" encourage students to try to articulate—and show in their representations—how their arguments apply to a class of numbers.

As they use representations such as this one to develop mathematical arguments, elementary students often *demonstrate* their ideas convincingly through manipulations of the representation before they can completely and clearly articulate the argument based on the representation. For many students at this age, their arguments will be a combination of representations or models, actions on these representations, and words describing the actions. Their final static representation may not capture their complete argument.

Finally, this example shows how students build on each others' arguments and representations and, as a class, consider to what numbers their ideas apply. Through their discussion, the teacher's questions, and their representations, students raise questions about whether any unit fraction can fit their ideas.